A simple shear-free non-singular spherical model with heat flux

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Abstract

We obtain an exact simple solution of the Einstein equation describing a spherically symmetric cosmological model without the big-bang or any other kind of singularity. The matter content of the model is shear free isotropic fluid with radial heat flux and it satisfies the weak and strong energy conditions. It is pressure gradient combined with heat flux that prevents occurrence of singularity. So far all known non-singular models have non-zero shear. This is the first shear free non-singular model, which is also spherically symmetric.

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Since Senovilla's discovery [1] of an exact singularity free cosmological solution of the Einstein equation representing a perfect fluid with the equation of state $\rho = 3p$ (and subsquently in the same framework the one [2] with $\rho = p$), it is now being recognised that the singularity theorems [3] can not, as generally believed earlier, prevent occurrence of non-singular cosmological solutions satisfying all the energy and causality conditions. And there is no conflict with the theorems in this. The theorems became inapplicable because one of the assumptions, existence of closed trapped surface, is not respected by these solutions and its violation does not entail any unphysical behaviour for the matter content. This assumption was however always a suspect [4] but this fact was not fully appreciated in absence of a non-singular solution. The Senovilla solution did this signal service of dispelling the folklore belief.

A large family of non-singular cosmological models [5] and its generalization with heat flux [2] has been considered but they are all cylindrically symmetric (see an excellent recent review [6]). For practical cosmology, spherical symmetry is however more appropriate. It is therefore pertinent to seek spherically symmetric non-singular models. The first model of this kind was obtained by one of us [7] which has imperfect fluid with heat flux (note, the expression for θ should have a negative sign before it) and it satisfies all the energy conditions and has no singularity of any kind. It was obtained by letting one of Tolman's solutions [8] expand. The solution has a free time function which can be chosen suitably to have non-singular behaviour for physical and kinematical parameters and there exist multiple such choices. It is also possible to have a non-singular model with null radiation flux [9]. These models are both inhomogeneous and anisotropic and have the typical behaviour, beginning with low density at $t \to -\infty$, contracting to high density at $t = 0$ and then again expanding to low density as $t \to \infty$. Nowhere any physical parameter diverges.

In the Raychaudhuri equation [10], which governs cosmological dynamics, it is acceleration (pressure gradient) and rotation (centrifugal force) that counteract gravitational collapse. In cosmology, there is absence of overall rotation and hence for checking collapse to avoid singualrity presence of acceleration becomes necessary. All the known non-singular models [1-2,5,7,9], are not only accelerating but also shearing. Though shear acts in favour of collapse in the Raychaudhuri equation but its dynamical action through

tidal acceleration makes collapse incoherent which acts against concentration of large mass in small enough region. This would ultimately work against formation of compact trapped surface.

Raychaudhuri [11] in one of his recent theorems establishes that the necessary condition for non-singular cosmological model is that space average of physical and kinematical parameters must vanish. That means the parameters must depend upon space variables. The space gradient of expansion is in vorticity-free spacetime given by space divergence of shear and heat flux [12]. Hence for non-singulaity presence of atleast one of them is necessary. The Ruiz-Senovilla family [5] of non-singular cylindrical models is the example of presence of shear without heat flux. It can be shown that presence of shear is in general essential for perfect fluid G-2 symmetric non-singular models [13]. The spherical non-singular model [7] has both shear and heat flux.

Then the question arises, could heat flux alone, of course combined with pressure gradient, avoid singularity? This is what we wish to demonstrate in this letter by obtaining a simple non-singular solution which describes an inhomogeneous shear-free spherical model filled with isotropic fluid and radial heat flux. The model satisfies the weak and strong energy conditions as well as has a physically acceptable fall off behaviour in both r and t for physical and kinematic parameters. Again there is a free time function which can be chosen suitably to give non-singular behaviour to model and there exist multiple such choices.

The metric of the model is given by

$$ds^2 = (r^2 + P)^{2n}dt^2 - (r^2 + P)^{2m}[dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2)] \qquad (1)$$

where

$$2n = 2m \pm \sqrt{8m^2 + 8m + 1}$$

in particular

$$2m = 1 - \sqrt{3/2} < 0, \ 2n = \sqrt{3/2}. \qquad (2)$$

Here $P = P(t)$ which can be chosen freely. The Einstein field equation for perfect fluid with radial heat flux reads as

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$$R_{ik} - \frac{1}{2}Rg_{ik} = -[(\rho + p)u_i u_k - pg_{ik} + \frac{1}{2}(q_i u_k + q_k u_i)] \tag{3}$$

where we have set $8\pi G/c^2 = 1$, $u_i u^i = 1 = -q_i q^i$, $q_i u^i = 0$, ρ, p denote fluid density and isotropic pressure, and q_i is the radial heat flux vector.

From eqns. (1) and (3) we obtain

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\begin{aligned}
\rho &= \frac{3m^2\dot{P}^2}{(r^2 + P)^{2n+2}} - 4m\frac{3P + (m+1)r^2}{(r^2 + P)^{2m+2}}, \\
p &= -\frac{m}{(r^2 + P)^{2n+2}}[2(r^2 + P)\ddot{P} + (3m - 2n - 2)\dot{P}^2] \\
&\quad + \frac{4}{(r^2 + P)^{2m+2}}[(m + n)P + n^2 r^2], \\
q &= \frac{4m(n+1)r\dot{P}}{(r^2 + P)^{n+2}}
\end{aligned}
\tag{4}
$$

and the expansion and acceleration are given by

$$\theta = \frac{3m\dot{P}}{(r^2 + P)^{n+1}}, \quad \dot{u}_r = -\frac{nr}{r^2 + P}. \tag{5}$$

We have freedom to choose the function $P(t)$ which could be chosen suitably to give non-singular behaviour to the above parameters. As a matter of fact there exist multiple choices, for instance $P(t) = a^2 + b^2 t^2, a^2 + e^{-bt^2}, a^2 + b^2 coswt, a^2 > b^2$. For all these choices it is clear that all the physical and kinematic parameters remain regular and finite for the entire range of variables. Note that it also admits an interesting oscillating behaviour in time in which the model oscillates between two finite regular states. The first case of oscillating non-singular model [14] was recently considered in the spherical family [7]. The oscillating non-singular models are quite novel and interesting of their own accord.

In non-oscillating case, all the parameters given above tend to zero as $r \to \infty$ or $t \to \pm\infty$. The universe begins with low density and contracts to maximum density and then again expands to low density without ever becoming singular. This is a typical behaviour for non-singular models [1,2,7]. However in the oscillating case, model oscillates in time between two regular finite states, and the parameters fall off to zero as $r \to \infty$. This is how

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oscillatory and non-oscillatory singularity free models differ from each-other in their global behaviour.

It is obvious from the simple expression for the metric that spacetime is causally stable. For verification of the energy contions, we will have to find the eigenvalues of the energy momentum tensor, which are given as follows:

$$[\frac{1}{2}(\rho - p + D), \ \frac{1}{2}(\rho - p - D), \ p, \ p], D^2 = (\rho + p)^2 - 4q^2. \qquad (6)$$

Note that in all the above expressions there would in view of eqn. (2) be relative dominance of the term of $(r^2 + P)^{-2(m+1)}$. The weak and strong energy conditions would require $\rho \geq 0, D \geq 0, \rho + p + D \geq 0, 2p + D \geq 0$. It can be easily verified that these conditions would hold good for the choices for $P(t)$ given above. The dominant energy condition which would require $\rho \geq p$ cannot however be satisfied as it is clearly violated for large r. Thus the model satisfies the weak and strong but not the dominant energy condition.

We have thus obatined a spherical model with isotropic pressure fluid and radial heat flux without the big-bang or any other kind of singularity. This is the first shear free non-singular model. It is inhomogeneous but isotropic. It is heat flux that combines with pressure gradient to avoid singularity. From the point of view of realistic cosmology, merits of the present model are its isotropy and spherical symmetry.

Apart fron the first Senovilla model [1] and a large family of cylindrical non-singular models [5,2], there now also exists a large family of spherical non-singular models [7,9] and to that the present one adds a novel family of shear free models. Even though it does not satisfy the dominant energy condition, it is a very simple and interesting model. It is remarkable to note that for the first time cosmic singularity has been avoided in absence of shear. There has been interesting cases of cosmological models [6], for instance [15], which do not satisfy all the energy conditions, yet deserve consideration for their other remarkable properties. The present model is simple, sheer free and isotropic and hence is interesting enough. Above all it is a very simple spherical model and thus also points to an important fact that non-singular cosmological solutions are no longer isolated but could occur more generally

even in spherical symmetry.

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